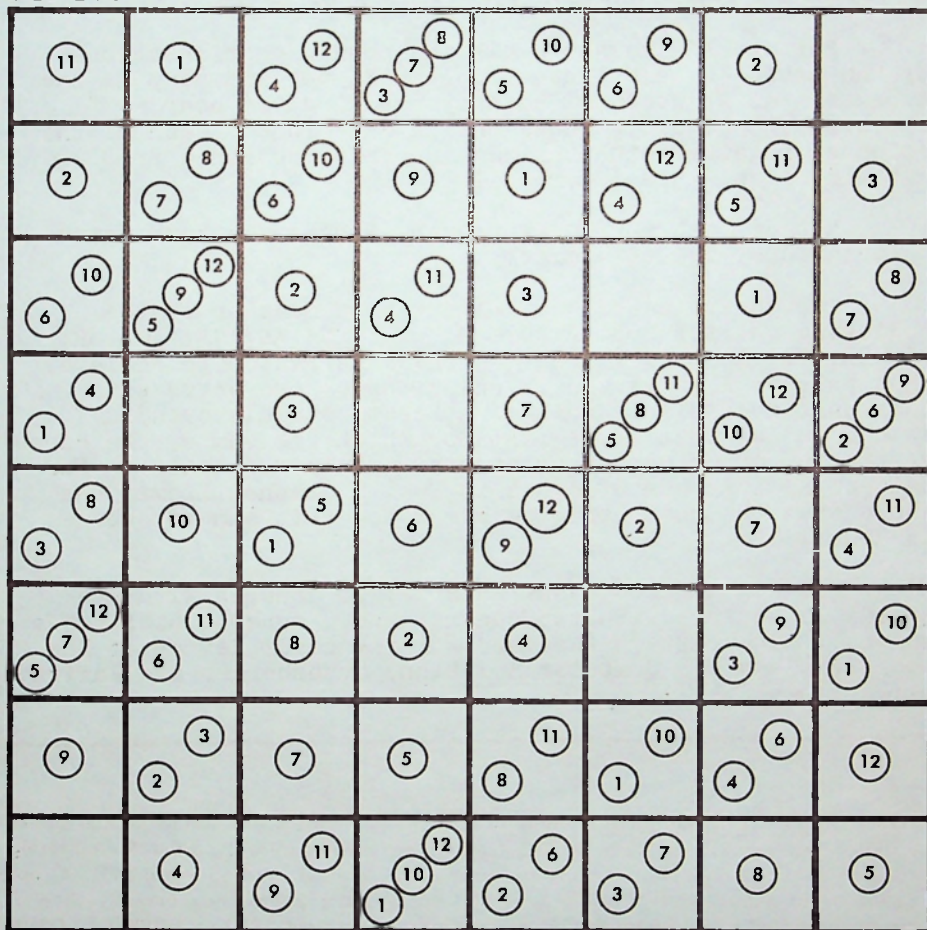


FIGURE K





## 8 Queens

The 8 queens problem requires placing 8 chess queens on a board in such a way that none of them is under attack from any of the others. There are just 12 solutions to that problem, and all of them are shown in Figure K. For example, all the circles labelled 7, taken in order from rows 1 to 8, lie in columns 4, 2, 8, 5, 7, 1, 3, and 6, and form one of the 12 solutions.

The array in Figure M also shows the 12 solutions. They are given in column A in such a way that the 4th square from the corner is listed first. (The circled 7's in pattern K represent row 7, column A of pattern M.)

Pattern M also shows all the possible orientations of the basic 12 solutions. Column E, for example, is the A pattern as it would be seen by rotating the board 90° clockwise. Figure P shows the orientation of each of the columns, together with the way it can be obtained from the pattern of column A numerically.

Notice that row 10 of pattern M has only four unique permutations, due to symmetry.

The entire pattern of Figure K is made up of the following entries from Figure M: 1C; 2E; 3D; 4F; 5G; 6H; 7A; 8A; 9E; 10G; 11F; 12B. There is reason to believe that pattern K shows the 12 solutions on one board with the least possible crowding. "Least possible crowding" is defined as that arrangement for which the sum of the cubes of the number of queens on each square is the least. For pattern K, this sum of cubes is 396. Stephen Lieman has shown that the theoretical minimum for this sum of cubes is 360.

Pattern K has 6 blank squares; 26 squares with one queen; 26 squares with two queens; and 6 squares with three queens. The pattern was used for the cover design of the book Problems for Computer Solution, Gruenberger and Jaffray, Wiley, 1965.

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This "least crowding" problem involves picking one set from each row of pattern M. There are

$$8^{11} \times 4 = 34359738368$$

ways to thread the pattern (there are only four possibilities at row 10). The scheme of attack used was as follows:

Two sets were arbitrarily selected from rows 1 and 2. These two sets were applied to a chessboard, and the resulting pattern was replicated in storage 8 times. Every set from row 3 was then added to these patterns, one set per pattern. The sum of cubes was calculated for the 8 patterns, and the one with the smallest sum was selected. That pattern was then replicated 8 times, and the 8 sets of row 4 were added, and the process was repeated, through row 12. This entire procedure was repeated for each possible choice of two sets from rows 1 and 2 ( $64$  such starting choices) and pattern K is the best result obtained. The effectiveness of the attack depends heavily on the ordering of the rows of Figure M, since we are taking a crude sample of the 34 billion possible choices.

Lieberman claims that it is possible to improve the situation, and achieve a pattern for which the sum of cubes will be less than 396. This, then, is the problem: find an arrangement for the 12 solutions to the 8 queens problem that will crowd the board less than pattern K.

PATTERN M

|    | A        | B        | C        | D        | E        | F        | G        | H        |
|----|----------|----------|----------|----------|----------|----------|----------|----------|
| 1  | 41582736 | 63728514 | 25713864 | 46831752 | 74286135 | 53168247 | 58417263 | 36271485 |
| 2  | 41586372 | 27368514 | 28613574 | 47531682 | 71386425 | 52468317 | 58413627 | 72631485 |
| 3  | 42586137 | 73168524 | 62713584 | 48531726 | 37286415 | 51468273 | 57413862 | 26831475 |
| 4  | 42736815 | 51863724 | 72418536 | 63581427 | 27581463 | 36418572 | 57263184 | 48136275 |
| 5  | 42736851 | 15863724 | 82417536 | 63571428 | 17582463 | 36428571 | 57263148 | 84136275 |
| 6  | 42751863 | 36815724 | 52814736 | 63741825 | 47185263 | 36258174 | 57248136 | 63184275 |
| 7  | 42857136 | 63175824 | 62714853 | 35841726 | 37285146 | 64158273 | 57142863 | 36824175 |
| 8  | 42861357 | 75316824 | 52617483 | 38471625 | 47382516 | 61528374 | 57138642 | 24683175 |
| 9  | 46152837 | 73825164 | 35714286 | 68241753 | 64285713 | 31758246 | 53847162 | 26174835 |
| 10 | 46827135 | 53172864 | 64718253 | 35281746 | 35281746 | 64718253 | 53172864 | 46827135 |
| 11 | 47526138 | 83162574 | 64713528 | 82531746 | 35286471 | 17468253 | 52473861 | 16837425 |
| 12 | 48157263 | 36275184 | 36814752 | 25741863 | 63185247 | 74258136 | 51842736 | 63724815 |

# 8 Queens Figure P

| Column | Orientation from Col. A                                  | Derivation numerically from column A                                                                                                                                                                                                                                                     |
|--------|----------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| B      | Reflect Col. H; that is, view pattern H from underneath. | Reverse the order of the digits in A.                                                                                                                                                                                                                                                    |
| C      | Reflect E.                                               | Each digit of C is formed by counting the position number (from the right) of the digits of B, as shown here:<br><br><div style="text-align: center;"> <p>Pattern 1B    6 3 7 2 8 5 1 4</p> <p>Positions<br/>numbered    8 7 6 5 4 3 2 1</p> <p>Form 1C       2 5 7 1 3 8 6 4</p> </div> |
| D      | Pattern A rotated 90° counter-clockwise.                 | Reverse the order of the digits of C.                                                                                                                                                                                                                                                    |
| E      | Pattern A rotated 90° clockwise                          | Take 9's complement of each digit of C.                                                                                                                                                                                                                                                  |
| F      | Reflect D.                                               | Reverse the order of the digits of E.                                                                                                                                                                                                                                                    |
| G      | Reflect A.                                               | Take 9's complement of each digit of A.                                                                                                                                                                                                                                                  |
| H      | Pattern A rotated 180°                                   | Take 9's complement of each digit of B.                                                                                                                                                                                                                                                  |

## Solution

Timothy Croy furnishes the following solution to Problem 41 (The Cubical Array) and thereby extends his subscription to POPULAR COMPUTING. The APL program shown below solves the problem and prints out the solution.

```

VCUBICALΔARRAY[[]]V
V Z+VCUBICALΔARRAY N
[1] Z+(3pN)pΔ4+7(1+ 100 100 100 T,(10000×1N)°.+(100×1N)°.+1N)*2
V

```



## Easter

The date of Easter is defined to be the first Sunday after the first Monday after the first full moon after the vernal equinox. The date for any year can be calculated by formulas developed by Gauss (taken from Elementary Number Theory by Uspensky and Heaslet, McGraw-Hill, 1939). The calculations for 1974 will be shown.

Call the year  $N$  and its century  $C$  ( $N = 1974$  and  $C = 19$  for this year). Calculate

$$M = 15 + C - \left[ \frac{C}{4} \right] - \left[ \frac{8C + 13}{25} \right]$$

where the brackets denote the greatest integer. For all of the current century,  $M = 24$ .

$$L = 4 + C - \left[ \frac{C}{4} \right] \bmod 7.$$

Again, in this century,  $L = 5$ .

$$\begin{array}{ll} a = N \bmod 4 & (\text{for } 1974, a = 2) \\ b = N \bmod 7 & (b = 0) \\ c = N \bmod 19 & (c = 17) \\ d = (19c + M) \bmod 30 & (d = 17) \\ e = (2a + 4b + 6d + L) \bmod 7 & (e = 6) \end{array}$$

Then, with certain exceptions, Easter is either

$$\text{March } (22 + d + e) \quad \text{or} \quad \text{April } (d + e - 9)$$

For 1974, these are March 45 and April 14 (really the same thing).

The exceptions (which occur only in 1954 and 1981 in this century) are these:

If  $d = 29$  and  $e = 6$ , Easter is April 19;

If  $d = 28$ ,  $e = 6$ , and  $M$  is one of 2, 5, 10, 13, 16, 21, 24, or 29, Easter is April 18.

Easter can thus be as early as March 22 or as late as April 25; neither of these extremes occurs in this century.

# Sieves

PROBLEM 43 A-H

The familiar Sieve of Eratosthenes yields prime numbers by the following algorithm:

Write down the consecutive integers from 2 to  $N$ . Circle the 2 and cross off every second integer following. Circle the next remaining integer (3) and cross off every third integer following. Continue this process: circle the next remaining integer ( $K$ ) and cross off every  $K$ th integer following (which will include integers previously crossed off). For the first few integers, the procedure then produces:

|               |      |               |               |               |      |               |              |               |               |
|---------------|------|---------------|---------------|---------------|------|---------------|--------------|---------------|---------------|
| (2)           | (3)  | <del>4</del>  | (5)           | <del>6</del>  | (7)  | <del>8</del>  | <del>9</del> | <del>10</del> | (11)          |
| <del>12</del> | (13) | <del>14</del> | <del>15</del> | <del>16</del> | (17) | <del>18</del> | (19)         | <del>20</del> | <del>21</del> |

The resulting list of (circled) prime numbers is correct to the square of the last  $K$  used.

Listed below are some other schemes for sieving the integers. In each case, the Problem is to determine the 1000th circled number.

1. Apply the same scheme as Eratosthenes, but cross off every  $K$ th remaining number. The resulting sequence begins 2, 3, 5, 7, 11, 13, 17, 23, 25, 29, 37,...

2. Apply the scheme of No. 1, but begin the series of integers with 3, rather than 2. The resulting sequence begins 3, 4, 5, 7, 8, 11, 13, 17, 19, 20, 26,...

3. Using the integers from 3 to  $N$ , when  $K$  is circled, cross off subsequent integers with the value  $KX+1$ . For example, when 11 is circled, cross off 12, 23, 34, 45, 56, and so on.

4. Using the integers from 7 to  $N$ , when  $K$  is circled, cross off subsequent integers with the value  $KX-1$ . For example, when 7 is circled, cross off 13, 20, 27, 34, 41, 48, and so on. The resulting sequence begins 7, 8, 9, 10, 11, 12, 14, 16, 18, 22, 25,...

5. Using the integers from 3 to  $N$ , when  $K$  is circled, cross off every 3rd remaining number. The resulting sequence begins 3, 4, 5, 7, 10, 14, 20, 29,...

6. Using the integers from 3 to  $N$ , when  $K$  is circled, cross off every  $M$ th remaining number, where  $M$  is 2, 4, or 5.

7. Using the integers from 2 to  $N$ , circle the 2 and cross off every second number (that is, all even numbers). Circle the first remaining number, 3, and



PC12-1

cross off every third of all remaining numbers (that is, cross off 5, 11, 17, 23, 29, and so on). Circle the first remaining number, 7, and cross off every seventh of all remaining numbers (that is, cross off 19, 39, 61, 81, 103, and so on). Those numbers finally remaining (including 1) form the sequence (1, 2, 3, 7, 9, 13, 15,...) that Ulam named Lucky Numbers.

8. Another of Ulam's sequences is

(1, 2, 3, 4, 6, 8, 11, 13, 16, 18, 26,...)

in which each new member can be formed in one and only one way by adding two different earlier numbers. Thus, numbers like 12 and 15 do not appear because they can be formed in more than one way ( $8+4$ ,  $11+1$ , for example), and 33 will not appear because it cannot be formed at all.

The original scheme of Eratosthenes for locating prime numbers is not, at first glance, a practical notion, since it seems to say "write down all the numbers in the desired range and then eliminate those that are not prime." Offhand, this corresponds to the advice to young sculptors on how to carve a beautiful statue of a horse: "Take this block of marble and cut away the parts that don't look like a horse." The process suddenly becomes practical with D. H. Lehmer's observation that the numbers themselves are not necessary to the scheme, but only the positions of the numbers, and a computer contains lots of positions; namely, bits. For example, a block of 1000 words (on a 32-bit word machine) can represent 32,000 consecutive numbers. Starting with these bits all set to zero, ones can be stored at every second position after position 2; at every third position after position 3, and so on, following the pattern laid down by Eratosthenes. The bit positions that are still zero after sifting, represent prime numbers. This was the procedure used in 1959 to validate the table of 6,000,000 prime numbers which had been calculated by a less efficient scheme. The entire check calculation took 21 minutes on an IBM 7094, where the original calculation, in 1957, consumed 120 hours on an IBM 704.

For contest purposes (such as the offer to undergraduates made in PC12) these 8 problems are subject to the following rules. (1) At least 4 of the sequences must be explored, and (2) the computer printout should show the first 100 numbers of the sequence, the 1000th number, and the limit on the value of N that was used in the sieve.

## BOOK REVIEWS

**A** THE ELEMENTS OF FORTRAN STYLE  
Charles B. Kreitzberg  
Ben Schneiderman  
Harcourt Brace Jovanovich, 1972, 120 pages.

**B** THE ELEMENTS OF PROGRAMMING STYLE  
Brian Kernighan  
P. J. Planger  
McGraw-Hill, 1974, 147 pages.

**C** FORTRAN TECHNIQUES  
A. Colin Day  
Cambridge University Press, 1972, 96 pages.

Three slim books are now available that all deal with programming style. All three are available in soft cover, 6 x 9 size. Anyone interested in producing better Fortran programs should own all three.

A and B both pay homage to The Elements of Style, W. S. Strunk, Jr., and E. B. White, Macmillan, 1959 (the current version of Strunk's 1919 classic) and point out the parallels between the rules of style in English prose and the need for applying those same rules to the writing of programs.

C is the most straightforward--it deals mostly with ways to perform stock jobs in neat and efficient, if not particularly elegant, Fortran code. Some of the topics covered are: flags and switches; DO-loops in a wide variety of uses; packing and unpacking numbers; number conversions; plotting on line printers; searching and sorting; list processing. C is a no-nonsense compendium of How to Do It.

Books A and B are concerned with style; that is, with programs that are not only correct, but that are easy to read, well documented, efficient, and readily modified. Book A sticks to Fortran; B throws in PL/I. A's emphasis is on the building of program modules; B analyzes longer programs.

Book B, the most recent of the three, is the most dramatic: all of its program examples are taken from standard textbooks. This is a bold idea indeed. Although the authors explain carefully that they do not wish to embarrass anyone, the whole book consists of criticism of programs that have been held out as models to students, in all the best-selling Fortran texts. The



program from the McCracken and Weinberg article in the October 1972 DATAMATION ("How to Write a Readable Fortran Program") is reproduced, together with the letters of comment on it in subsequent issues, and a counter example that the authors found for which the program doesn't even work properly. Since book B displays other authors' codes as bad examples, it should be impeccable itself. Thus, non-ANSI Fortran usage in two of its programs is a weakness.

Book B has little inserted homilies scattered throughout, which contain much condensed wisdom, such as:

Make sure your code "does nothing" gracefully.  
 Don't diddle code to make it faster--find a better algorithm.  
 Make it right before you make it faster.  
 Make it fail-safe before you make it faster.  
 Make it clear before you make it faster.  
 Don't comment bad code--rewrite it.  
 Don't just echo the code with comments--make every comment count.

Book A uses real printouts, reproduced photographically; B uses typesetting for its codes. At one of the few places that A uses typesetting (page 80), two of the exponents are wrong. Book A has some curious references in its bibliography, such as Donn Parker on Ethics and Alan Westin on Privacy. A has an inadequate index; B has an excellent index.

In book B, the authors explain why their improvements to textbook codes really improve things. For example, in completely revamping a code for calendar manipulation, they conclude:

"We have reduced the fifteen internal labels to one, and the 26 GOTOs to two. Since a branch can occur only to a label, we have that much less to worry about." There follows a sensible rule on GOTOs: Use GOTOs only to implement a fundamental structure.

Among other criticisms of the codes used as bad examples, B notes obvious typos in the texts. This may seem like dirty pool (few books--and this may include book B, although none were found--are completely free of typos). The justification is given, on page 32:

"...regrettably, a program with a typo in it won't work. If you're lucky, it will fail to compile. Worse, like this one, it may run but provide subtly wrong answers."



These books deal with the niceties of style in programming. Book A has exactly one flowchart, which contains a decision box containing the phrase "All Equal" with its exits labelled "yes" and "no." Professor Strunk would probably have insisted that an English question end with a question mark. It's a nitpicking matter, but in a book that harps on style, it would be well for the authors to get their style above criticism.

All three of these books are excellent, and can stand re-reading from time to time by anyone who honestly wants to improve his coding ability. Except for book C, the choice of language is immaterial; the principles of writing good code cut across all languages.

## Credibility Game

Associate Editor David Babcock furnishes the following statistics on 10,000 plays of the Credibility Game (PC10, Problem 33).

| MOVES | FREQ | MOVES | FREQ | MOVES | FREQ | MOVES | FREQ |
|-------|------|-------|------|-------|------|-------|------|
| 8     | 1    | 21    | 327  | 34    | 83   | 47    | 24   |
| 9     | 87   | 22    | 301  | 35    | 75   | 48    | 21   |
| 10    | 373  | 23    | 277  | 36    | 70   | 49    | 9    |
| 11    | 752  | 24    | 245  | 37    | 57   | 50    | 13   |
| 12    | 843  | 25    | 238  | 38    | 46   | 51    | 17   |
| 13    | 887  | 26    | 210  | 39    | 41   | 52    | 16   |
| 14    | 760  | 27    | 149  | 40    | 48   | 53    | 10   |
| 15    | 702  | 28    | 159  | 41    | 42   | 54    | 6    |
| 16    | 572  | 29    | 124  | 42    | 41   | 55    | 8    |
| 17    | 551  | 30    | 134  | 43    | 46   | 56    | 10   |
| 18    | 419  | 31    | 125  | 44    | 34   | 57    | 5    |
| 19    | 392  | 32    | 107  | 45    | 23   | 58    | 5    |
| 20    | 350  | 33    | 92   | 46    | 16   | 59    | 6    |

The longest game ran 95 moves. The 51 games missing from the table had 6 or less moves each.



|       |       |       |       |       |       |       |       |
|-------|-------|-------|-------|-------|-------|-------|-------|
| ABOVE | CORPS | EVENT | HOVER | LIVER | PAPER | RULER | SPORT |
| ACUTE | COVET | EVERY | ICONS | LOOSE | PARTY | RUSTY | SQUIB |
| ADAPT | CRAFT | FAULT | IDEAL | LOSER | PHOTO | SHAFT | SQUID |
| ADEPT | CRANE | FEAST | IMAGE | LOYAL | PLAID | SHAKE | STOCK |
| ADOPT | CREED | FIELD | INCUR | LUCID | PLATE | SHALL | STRIP |
| ADULT | CRIMP | FIFTH | INEPT | LURID | PLEAD | SHARE | STUCK |
| AFTER | CRISP | FIGHT | INGOT | MANIC | PLUCK | SHELF | SUGAR |
| AGONY | CROCK | FINAL | INDEX | MATCH | PRIDE | SHIFT | TABLE |
| AGREE | CROON | FRAME | INNER | MERRY | PRINT | SHORE | TACIT |
| ALBUM | CROSS | FROST | INPUT | MIGHT | PROOF | SIGHT | TAINT |
| ALIVE | CRUDE | FRUIT | IRATE | MINER | PYLON | SIXTH | TENTH |
| ALONE | DANCE | FUDGE | ISSUE | MISTY | QUEEN | STACK | TEPID |
| ALOOF | DEBIT | GLOOM | IVORY | MOTOR | QUEER | STAIN | TIMER |
| ANISE | DEBUG | GLOVE | JAUNT | MUSIC | QUEST | STAKE | TIMID |
| ASHEN | DEIGN | GNOME | JEWEL | NADIR | QUEUE | STALK | THEME |
| BAKER |       |       |       |       |       |       | THING |

In this game, each of two players starts with a 5 x 5 array of squares. The players take turns inserting letters into both arrays. Each player must use all letters, and the players do not see each other's array. The object of the game is to build words: horizontally, vertically down, and diagonally northwest to southeast or southwest to northeast. Scoring is done when the arrays are filled. 5-letter words count 10; 4-letter words count 5; 3-letter words count 1. The same word cannot be used twice in a pattern, and plurals count. The strategy of the game calls for a player having some pattern in mind, but to be able to change that pattern to accomodate the letters furnished by the opponent. Erasures are not permitted; each letter must be applied at the time it is called. The theoretical maximum is 120 points. It is fairly easy to produce patterns in the 70's, such as:

|       |       |       |
|-------|-------|-------|
| RADAR | ICONS | ABOVE |
| OBESE | DOUBT | NYLON |
| UMBRA | ELITE | INGOT |
| GRAND | ABOVE | SUGAR |
| HEAVY | LOYAL | EMPTY |

**Guggenheim**

PROBLEM 44

Problem: given a supply of 5-letter words, to select them in groups of 5 to form suitable patterns. Write a program to select words from the list given here, to achieve patterns of more than 80 points.

|       |       |       |       |       |       |       |       |
|-------|-------|-------|-------|-------|-------|-------|-------|
| CHOSE | DRAIN | GOOSE | JOINT | NAVAL | QUICK | STALL | WEIRD |
| CIDER | DREAM | GRAND | JOULE | NEVER | RABID | STAMP | WHILE |
| CITED | DRINK | GRATE | JUDGE | NIGHT | RADAR | STAND | WORRY |
| CIVIL | DROSS | GREAT | JUICE | NINTH | RANGE | STEAL | WORST |
| CLASS | EAGLE | GREEN | JUMBO | NOOSE | RAPID | STEAM | WOULD |
| CLEAN | EARTH | GRIPE | KINGS | NORTH | RAZOR | STEEL | WOUND |
| CLEAR | EBONY | GROWL | KITTY | NOVEL | READY | STICK | WRING |
| CLIMB | EIGHT | GRIME | KNACK | NYLON | REIGN | STILL | WRITE |
| CLIME | ELDER | GRUNT | KNAVE | OBESE | REPLY | STING | WRONG |
| CLING | ELITE | GUSTY | KNEAD | OLIVE | RIGHT | SLIDE | YEARN |
| CLOSE | EMERY | HATCH | KNIFE | ORBIT | ROAST | SLIME | YEAST |
| LOVE  | EMPTY | HASTY | KNOCK | ORDER | ROSIN | SLING | YIELD |
| CODER | ENTER | HAVEN | KRAFT | ORGAN | ROTOR | SMILE | YOUNG |
| COLOR | ENTRY | HEART | LATCH | OTTER | ROUGH | SOUND | YOUTH |
| COMIC | ENVOY | HEAVY | LIGHT | OUGHT | ROUND | SOUTH | ZEBRA |

## 3X+1 Strings

Various aspects of the  $3X+1$  Problem have been discussed (PC1-1 and PC4-6). In this problem, take a positive integer,  $N$ , let  $X$  equal  $N$ , and follow the algorithm:

Replace  $X$  by  $X/2$  if  $X$  is even.

Replace  $X$  by  $3X+1$  if  $X$  is odd.

Stop when  $X$  equals 1.

Call the number of terms so generated  $A$ , counting the original number.

It has been noted that consecutive values of  $N$  can converge with the same  $A$  value. For example, the numbers 98 through 102 all have an  $A$  value of 26; the numbers 943 through 949 all have an  $A$  value of 37, and so on. The calculations for the numbers 98 through 102 are shown here:

|     |       |     |      |     |                   |
|-----|-------|-----|------|-----|-------------------|
| 98  | 99    | 100 | 101  | 102 | ← N and initial X |
| 49  | 298   | 50  | 304  | 51  |                   |
| 148 | 149   | 25  | 152  | 154 |                   |
| 74  | 448   | 76  | ← 76 | 77  |                   |
| 37  | 224   | 38  |      | 232 |                   |
| 112 | ← 112 | 19  |      | 116 |                   |
| 56  |       | 58  | ← 58 |     |                   |
| 28  |       | 29  |      |     |                   |
| 14  |       | 88  |      |     |                   |
| 7   |       | 44  |      |     |                   |
| 22  | ← 22  |     |      |     |                   |
| 11  |       |     |      |     |                   |
| 34  |       |     |      |     |                   |
| 17  |       |     |      |     |                   |
| 52  |       |     |      |     |                   |
| 26  |       |     |      |     |                   |
| 13  |       |     |      |     |                   |
| 40  |       |     |      |     |                   |
| 20  |       |     |      |     |                   |
| 10  |       |     |      |     |                   |
| 5   |       |     |      |     |                   |
| 16  |       |     |      |     |                   |
| 8   |       |     |      |     |                   |
| 4   |       |     |      |     |                   |
| 2   |       |     |      |     |                   |
| 1   |       |     |      |     |                   |

For the numbers 99 through 102, it is not necessary to continue the calculation, since convergence with 26 terms can be seen.

This string phenomenon appears to hold true at any level. For example, the numbers from 996000315 through 996000323 all converge in 256 terms. The largest known



string is of length 40, as given in the table below.

A sub-problem involved within the  $3X+1$  String Problem is this: What is the logic of detecting and counting the strings, as successive A's are produced? That is, given a program that tests successive values of  $N$  for convergence by the  $3X+1$  algorithm, to note when successive A values are alike and print out the appearance of ever-larger strings. A flowchart for this logic is needed.

PROBLEM 45

In the following table, strings are listed. In each case, the value of  $N$  given is the last of the string; for example, the numbers 9089, 9090, 9091, 9092, 9093, 9094, and 9095 all have the same number of terms to convergence. These are not the first appearances of strings of the stated length in all cases.

| String<br>length | Final<br>N | String<br>length | Final<br>N |
|------------------|------------|------------------|------------|
| 3                | 9006       | 18               | 137169     |
| 4                | 9037       | 19               | 447262     |
| 5                | 9062       | 20               | 454461     |
| 6                | 9077       | 21               | 152216     |
| 7                | 9095       | 22               | 212181     |
| 8                | 9463       | 23               | 362520     |
| 9                | 1688       | 24               | 221208     |
| 10               | 240763     | 25               | 57370      |
| 11               | 358042     | 26               | 393242     |
| 12               | 131917     | 27               | 252574     |
| 13               | 132429     | 29               | 331806     |
| 14               | 3000       | 30               | 524318     |
| 15               | 134270     | 32               | 913350     |
| 16               | 243839     | 35               | 1032909    |
| 17               | 7099       | 40               | 596349     |

# Overview of Computer Graphic Films

JOHN G. SCOTT

In viewing computer graphics from an artistic, rather than a technological, standpoint, it is possible to overlook some impressive examples of computing. For example, Computer Image Corporation has developed a marvelous method for manipulating prepared imagery, but has yet to produce a film anywhere near the beauty of some of Hy Hirsh's decade-old films. ENERI and DIVERTISSEMENT ROCOCO utilize very simple oscilloscope patterns, but are impressive, nonetheless.

Indeed, abstract and collage film artists are way ahead of their colleagues who work with computers, if only for Jordan Belson's ALLURES, the many films by the late Oskar Fischinger (ALLEGRETTO, MOTION PAINTING, etc.) and the optical printer masterpiece by Pat O'Neill, RUNS GOOD.

Lillian Schwartz and Ken Knowlton are the most advanced, visually, of the computer film makers. ENIGMA is a glorious, multi-layered color experience, while GOOGOLPLEX is their best in black and white. Fast paced and well edited, ENIGMA could only be improved with a more lyrical sound track.

Doris Chase's CIRCLES, CIRCLES/VARIATION 2, and SQUARES, leave much to be desired. These are computer films for people who detest computer films, appealing only to those who enjoy music in elevators and stores, which is produced by people who hate music.

CIBERNETIK 5.3 by John Stehura, is visually very interesting in that it combines live action photography with computer generated designs, but it is not well structured.

John and James Whitney have been the recognized leaders in this field for many years and their offspring, Michael and John, Jr. seem capable of joining them. James Whitney's LAPIS and YANTRA both rely to some degree on hand animation techniques (YANTRA also is hand solarized), so they are not as hard-edged as other computer films. YANTRA lacks a good score, but both films are quite intricate and very lovely.

John Whitney's EXPERIMENTS IN MOTION GRAPHICS demonstrates visually, and through lucid narration (by Whitney) many of the techniques involved in producing computer films. His PERMUATIONS delightfully introduces many different types of patterns, then proceeds to combine them effortlessly, one atop the next. It is his MATRIX which is the finest film, however, because of the combination of motion picture technique (color saturation and editing) and the excellent graphic design. A perfect sound track adds to its beauty.

The finest work appears to result when the film artist utilizes the computer as a tool with which to produce a film rather than as an end in itself--which is as it should be.

All films except GOOGOLPLEX are available from either

Creative Film Society  
7237 Canby  
Reseda, CA 91335

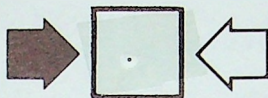
or

Pyramid Films  
Box 1048  
Santa Monica, CA 90401



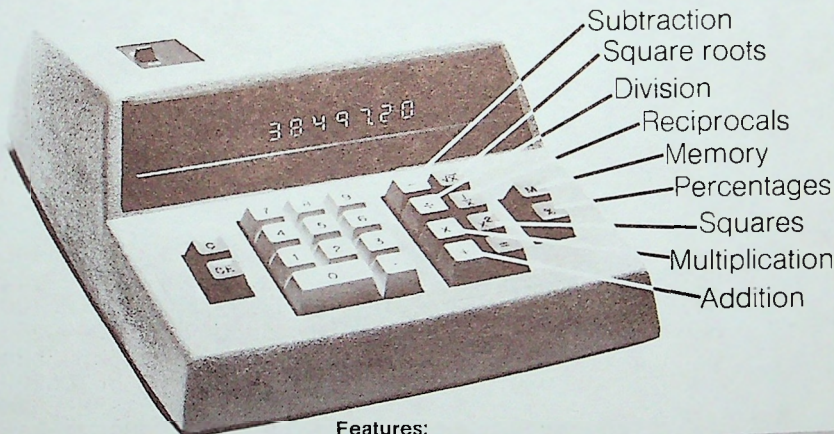
# Magnetic Cores

Attached to each copy of the initial press run of this issue of POPULAR COMPUTING is one or more 20-mil magnetic cores



Although the use of magnetic cores for computer storage is on the way out, it is still the most-used medium. The cores included here are relatively large; they are made as small as 14 mil (.014 inch outside diameter).

## **MITS Presents** **The new 908DM, Desk-Top Calculator.**



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\* Plus the option of programmability

### \*Prices: 908DM

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1. A constant
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3. An accumulator

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\*Combination 908DM and Programmer Kit \$299.95 Assembled \$399.95

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\*Prices subject to change without notice. Available from your local Olson Electronics Dealer

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505/265 7553 Telex Number 660401

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 or MasterCard \_\_\_\_\_  
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 (Include \$5.00 for Postage and Handling)  
☐ Model 908DM ☐ Programmer ☐ 908DM & Programmer  
 Please Send Information on Entire MITS Line

NAME \_\_\_\_\_  
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 CITY \_\_\_\_\_  
 STATE & ZIP \_\_\_\_\_  
 MITS/ 6178 Linn N.E. Albuquerque New Mexico 87108 505/265 7553 Telex # 660401

OW-W-74



## N-Series

|                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Log 13           | 1.1139433523068367692065051579423284308297291883870<br>6827180119097499755309163019424080764745425889966                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| Ln 13            | 2.5649493574615367360534874415653186048052679447602<br>0711641904551066346466732441017939957466344048949                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| $\sqrt{13}$      | 3.6055512754639892931192212674704959462512965738452<br>4621271045305622716694829301044520461908201849072                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| $\sqrt[3]{13}$   | 2.3513346877207574895000163399569145269158419834621<br>7510504025431158834268099656684980791160420284406                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| $\sqrt[5]{13}$   | 1.6702776523348103948036528913127146312910688456900                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| $\sqrt[7]{13}$   | 1.4425629194429777308794407657419497409423003367515                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| $\sqrt[10]{13}$  | 1.2923922207808318415109131988566871606210629646728                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| $\sqrt[100]{13}$ | 1.0259812724144340113583202244325547309032394760351                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| $e^{13}$         | 442413.39200892050332610277594908828178439130606058<br>97155723593307090218623364382122804522731305                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| $\pi^{13}$       | 2903677.2706132834049885961994878031304704718659429<br>0961329998720176675486201804780251979710340                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| $\tan^{-1} 13$   | 1.4940244355251185800019995584923659339919735025110<br>8078618204515146450708532493605153214314798911915                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| $13^{100}$       | 247933511096597253351107288473486513623877446787494<br>114981218909940615869983797556015828566293998218019<br>2171594001                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| $13^{1000}$      | 877712547297351164963075005029518868335143011009791<br>587625089497842979736915596129032182962500492014175<br>841671906680564557971074429054133768011377267004038<br>686384928365307832441547181678860494588909492578490<br>088581272498408784373744211192641381824585436261301<br>805877436870397160492185802311666558635887061294420<br>939805656260456124885992634433559882281588585109669<br>822677505315340332078243998767997832128953764564516<br>376725139675951980560309033269444955337153057161335<br>231100635005821798250973836208309492064945212335171<br>736633741024385365911331554758487165547991443921952<br>015717472913074635105907520740786601257438672606419<br>699286562714956623804462577907818662434718390591335<br>771885053705857808493288056970124259866314991127635<br>712535585079207363553367654125053108675737736996250<br>697937833721641118834776190100646081341350586146126<br>754572359046862785420203445056958162664893406219871<br>836230342028155588639455813740815945310339591878362<br>571321331435053105131255173302162715308107508014068<br>060808052973697565878622736225163272500943586654761<br>359875358470545595541969660928205919103196260416924<br>2974038517575645939316377801594539335940001 |